Optimality and Bifurcations in a Model for Collective Motion

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The problem

Who cares about collective robotic motion?

Agents copying each other's behavior arises in nature (allelomimesis—a cool word!)

Most robotic motion planning relies upon agents parsing others' position or orientation

How can allelomimesis be used in robotic motion planning?

Justh and Krishnaprasad 2015 Model

Coupling constant $\mu \epsilon \mathbb{R}_{\geq 0}$

minimize $$\int_0^1 \frac{1}{2}{u_1}^2 + \frac{1}{2}{u_2}^2 + \frac{1}{2}\mu(u_1 - u_2)^2 \, \mathrm{d}t$$

subject to

$$\dot{x}_1 = \cos(x_3) \quad \dot{x}_4 = \cos(x_6)$$
$$\dot{x}_2 = \sin(x_3) \quad \dot{x}_5 = \sin(x_6)$$
$$\dot{x}_3 = u_1 \quad \dot{x}_6 = u_2$$

$$\vec{x}(O) = \vec{x}_O$$
$$\vec{x}(1) = \overrightarrow{x_f}$$

u_i is the turning rate of robot *i*

Only studied for coupling of 0 or ∞!

Give us an optimal path for robot 1

Make the two robots' paths as similar in curvature as the coupling dictates

$$\int_0^1 \frac{1}{2}{u_1}^2 + \frac{1}{2}{u_2}^2 + \frac{1}{2}\mu(u_1 - u_2)^2 \, \mathrm{d}t$$

Give us an optimal path for robot 2

$$\int_0^1 \frac{1}{2}{u_1}^2 + \frac{1}{2}{u_2}^2 \quad + \quad \frac{1}{2}\mu(u_1 - u_2)^2 \quad \mathrm{d}t$$

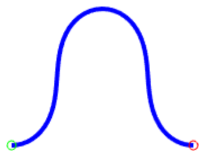
If μ is small, each robot will act more independently.

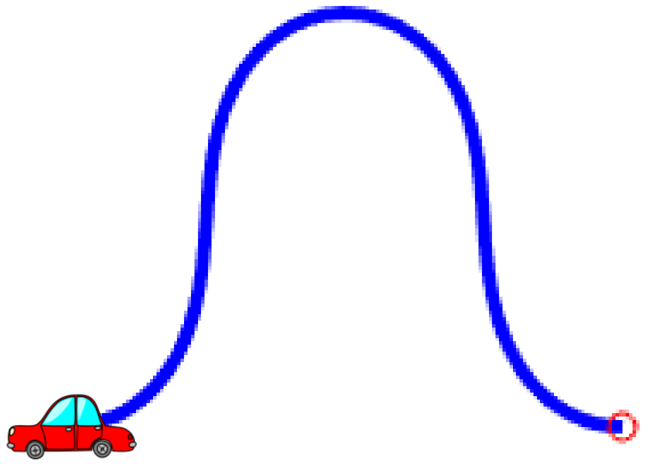
$$\int_0^1 \frac{1}{2}{u_1}^2 + \frac{1}{2}{u_2}^2 \quad + \frac{1}{2}\mu(u_1 - u_2)^2 \mathrm{d}t$$

If μ is large, the robots will attempt to follow similar paths.

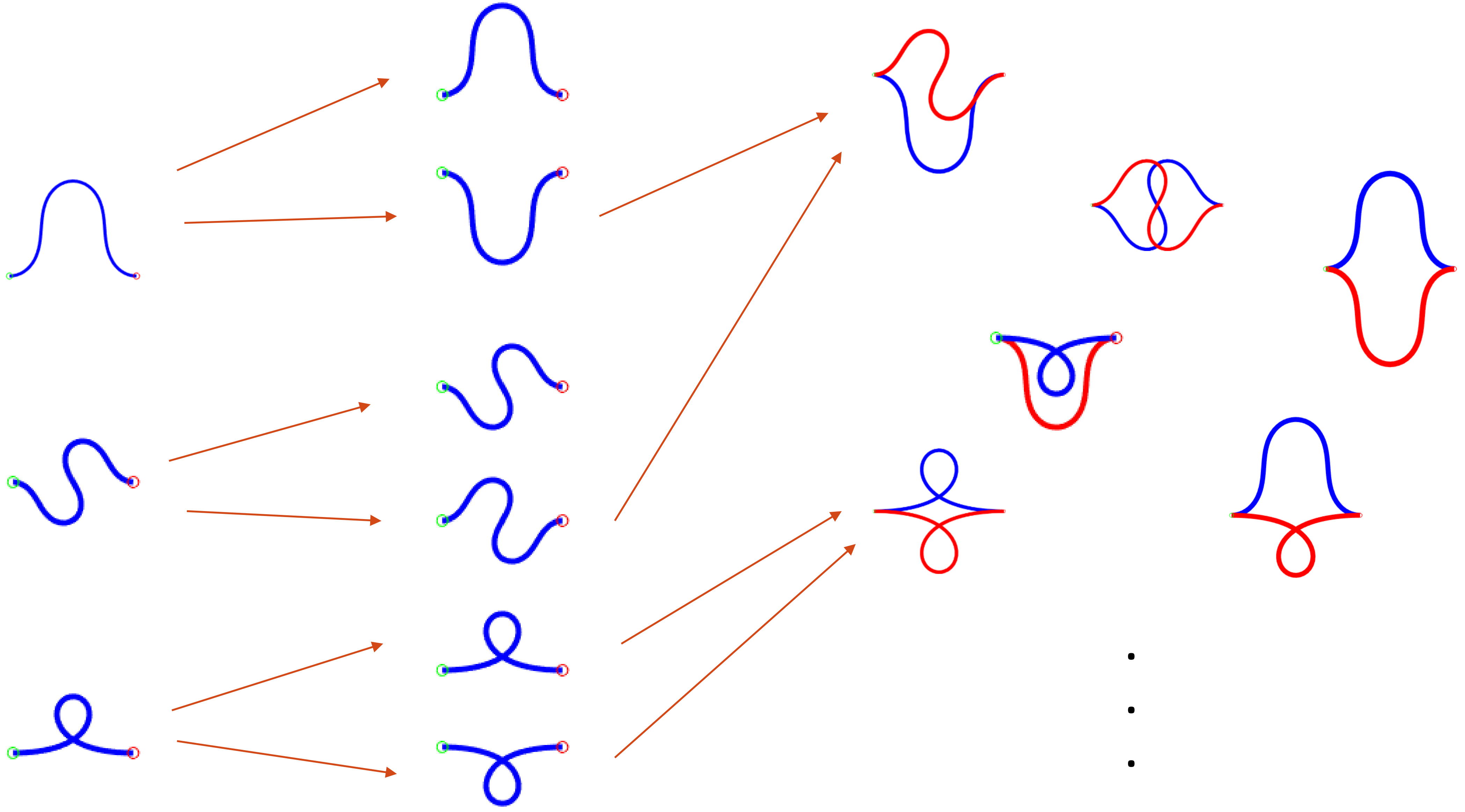
If $\mu = \infty$, the robots will follow the **same** path.

Canonical solution forms

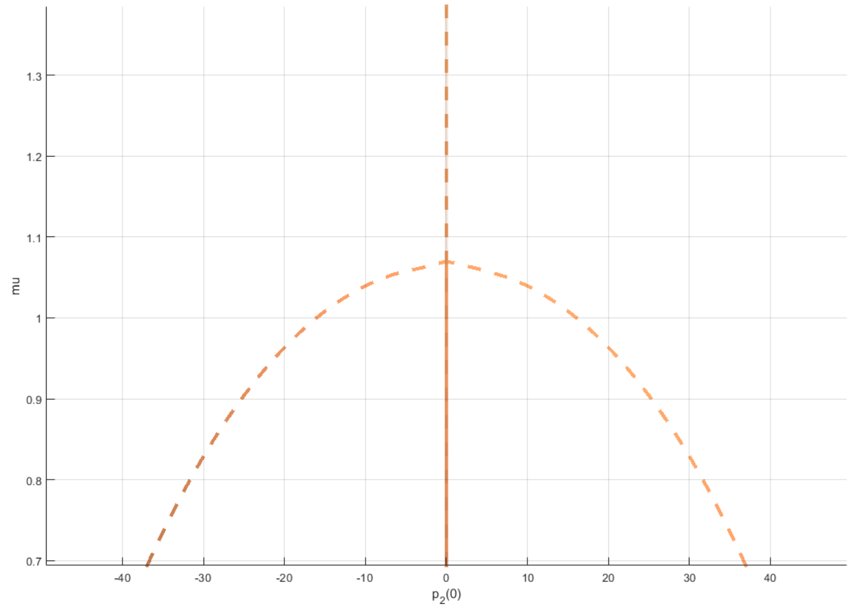
	Bell 2 inflection points Optimal
	S shape 3 inflection points Suboptimal
	Loop 0 inflection points Optimal



- Each robot starts at the origin (green circle) and ends at time $t = 1$ at $(0.5, 0)$ (red circle)
- More complex solutions can be described in terms of simple ones (hence, canonical)

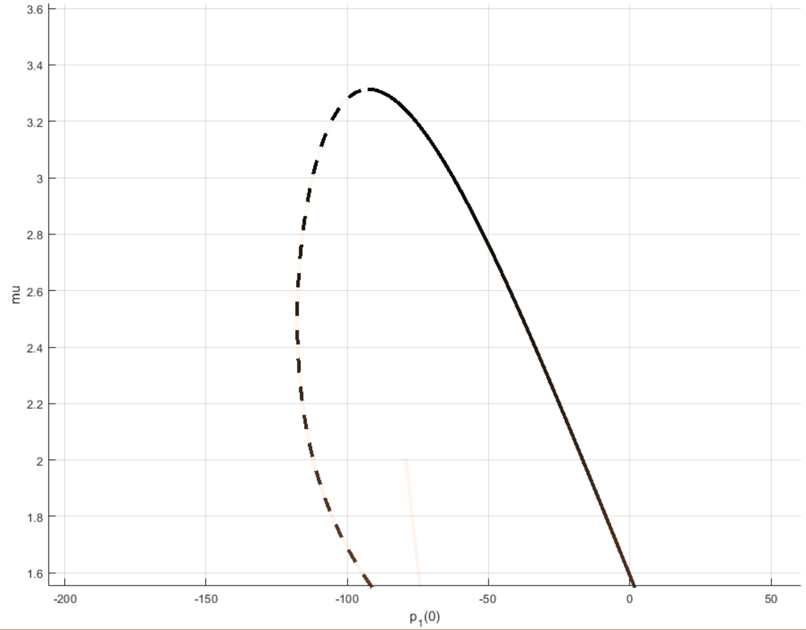


Bifurcations in the solution space



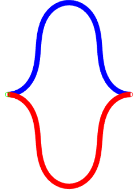
Pitchfork Bifurcation (subcritical)

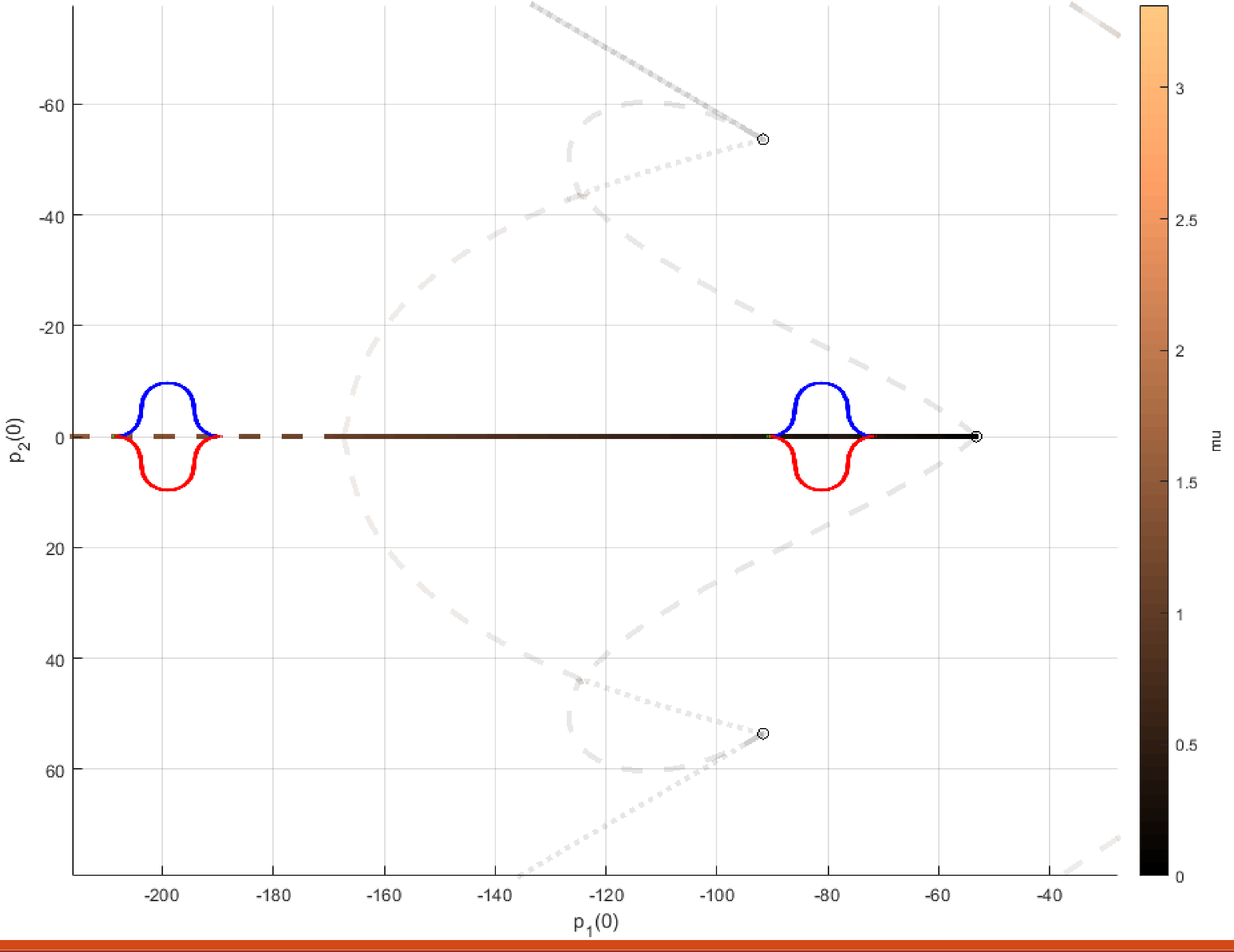
A suboptimal solution splits off into two suboptimal solutions one optimal solution



Fold (Saddle-node) Bifurcation

Two solutions—one optimal, one suboptimal—collide and obliterate each other





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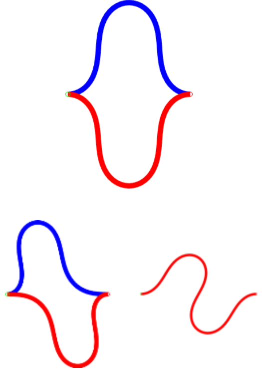

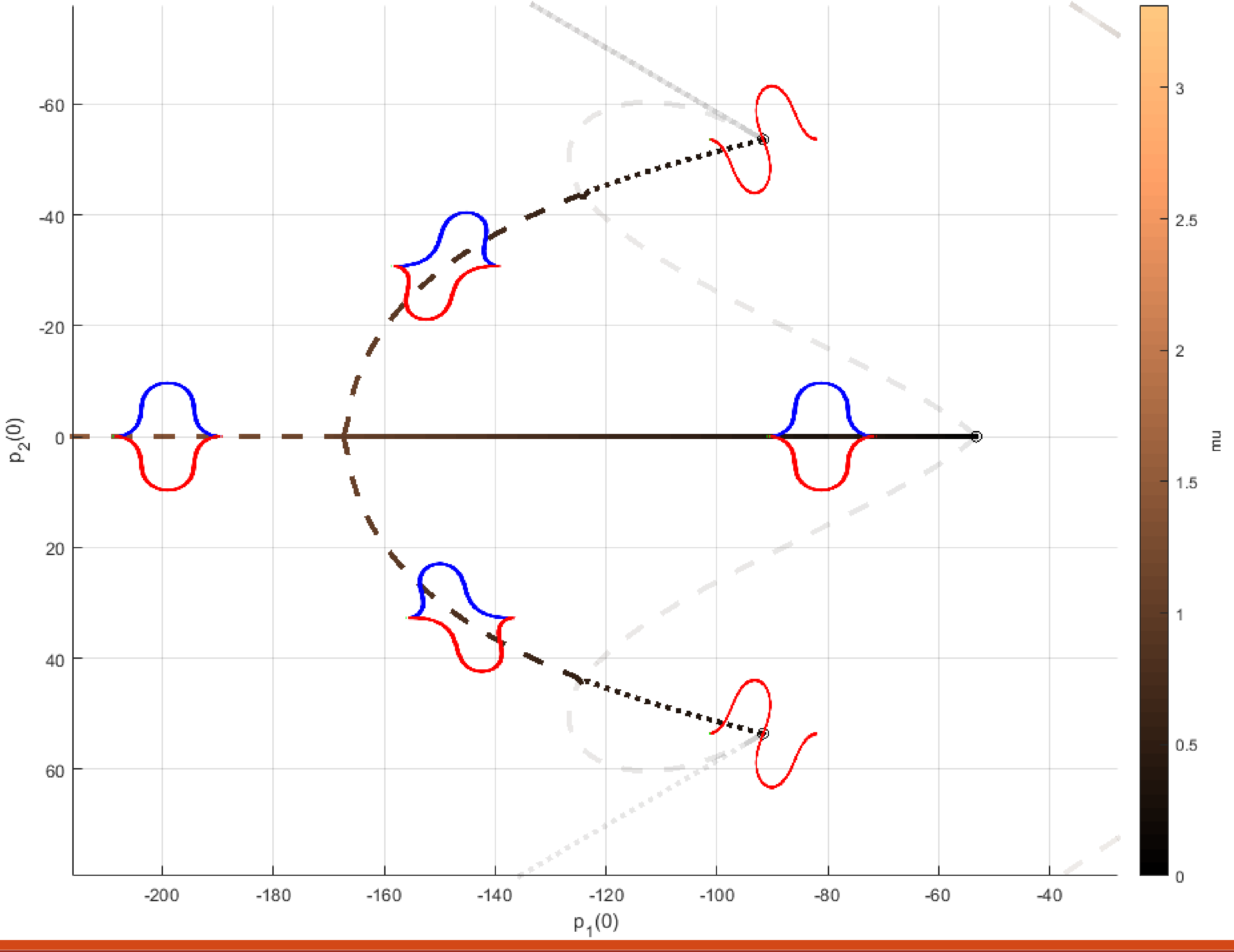
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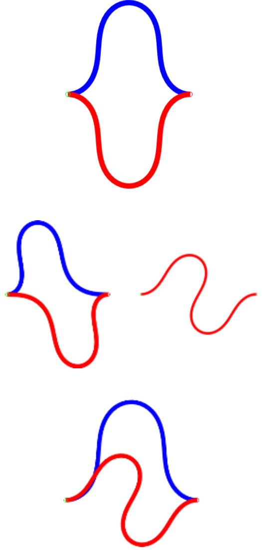

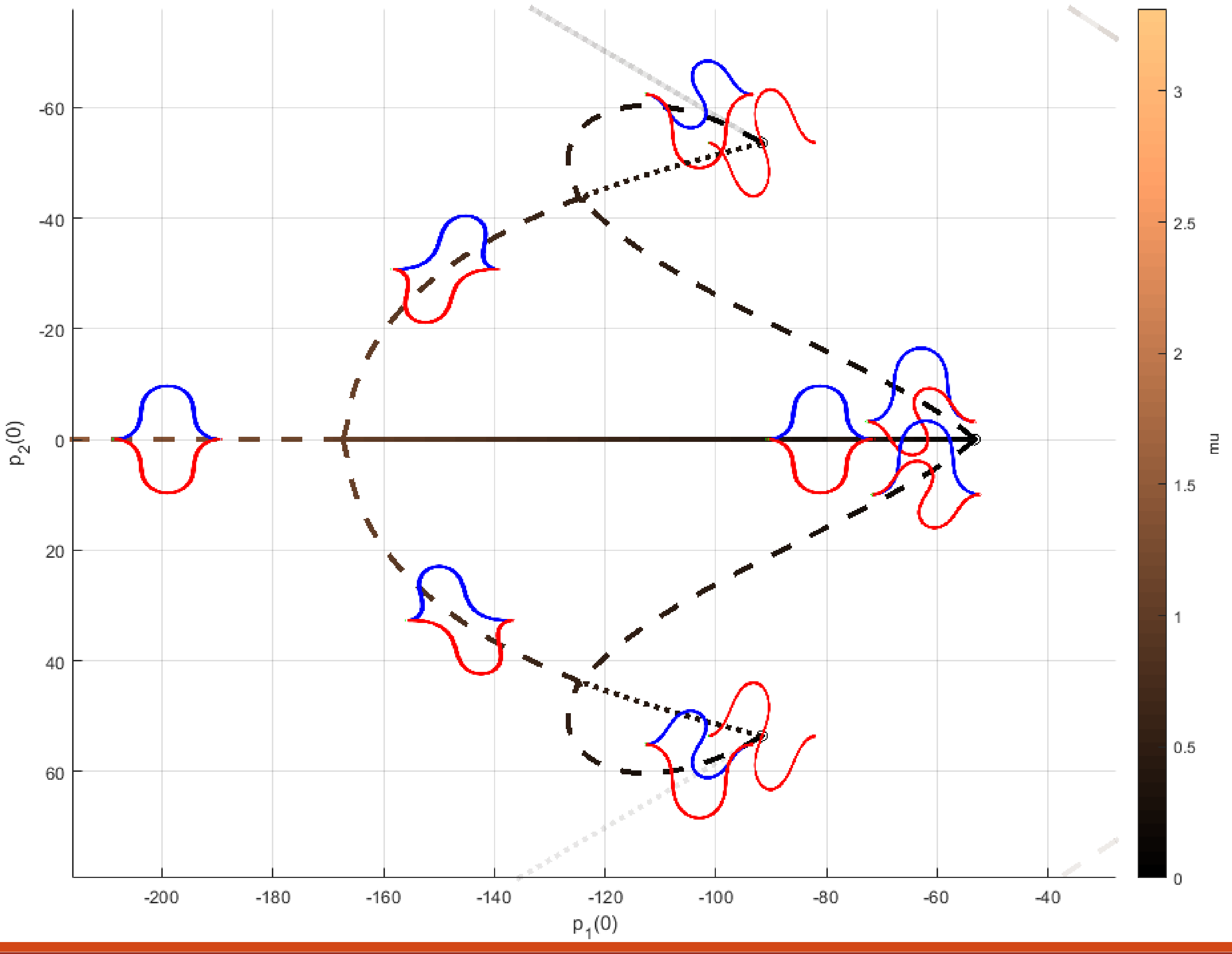

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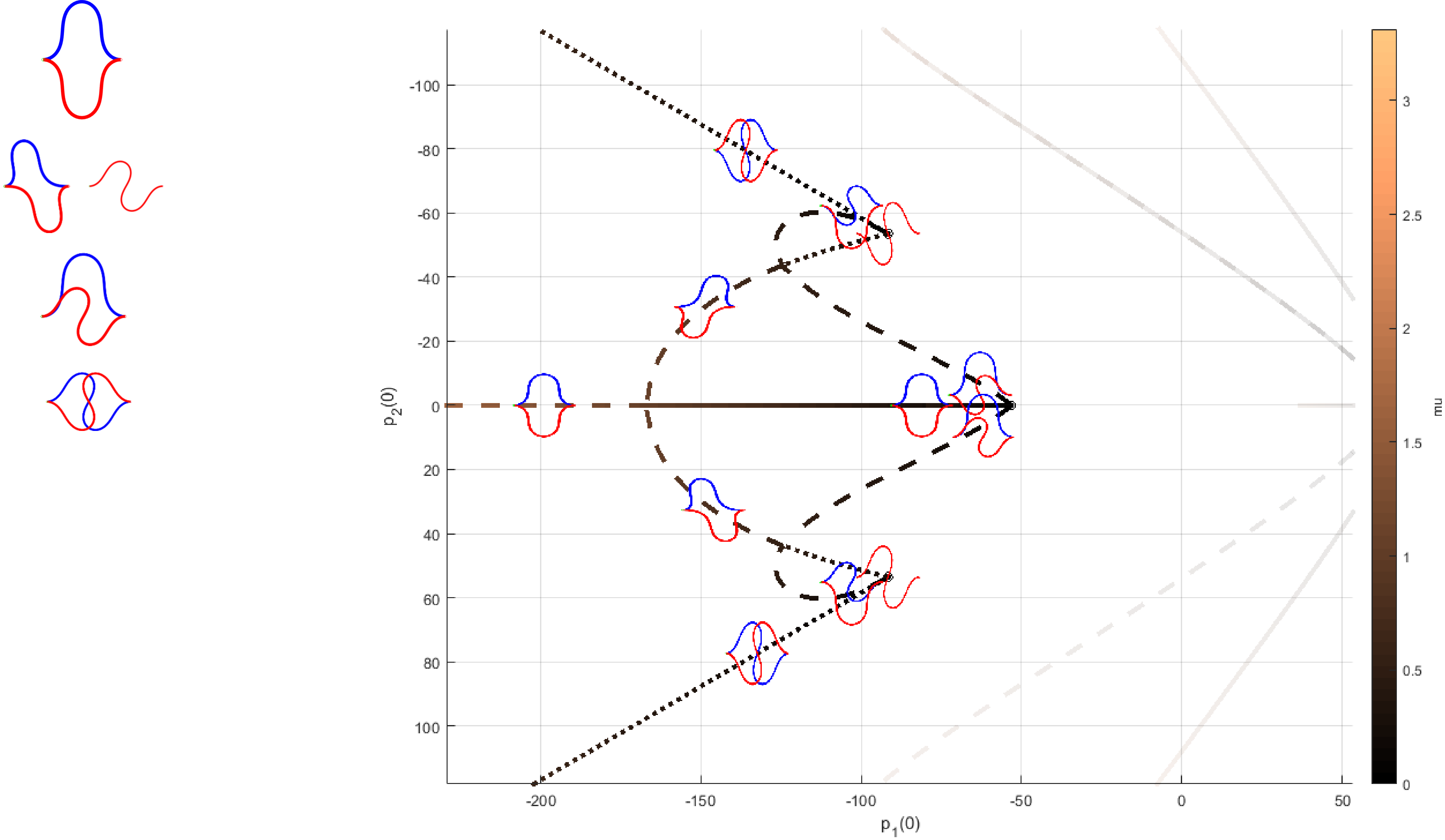
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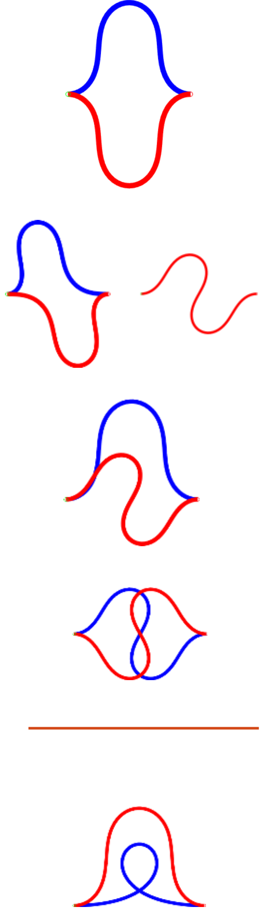

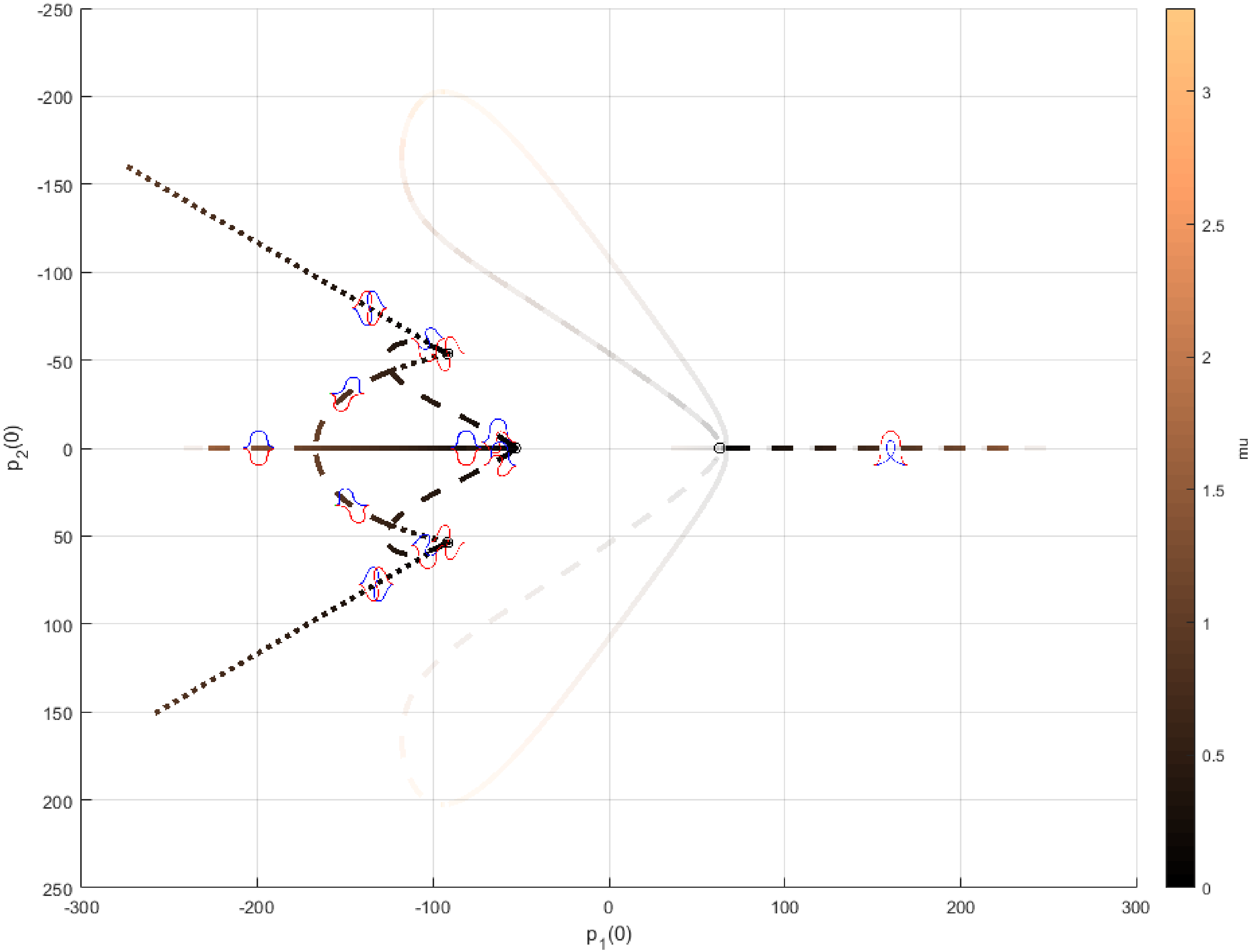
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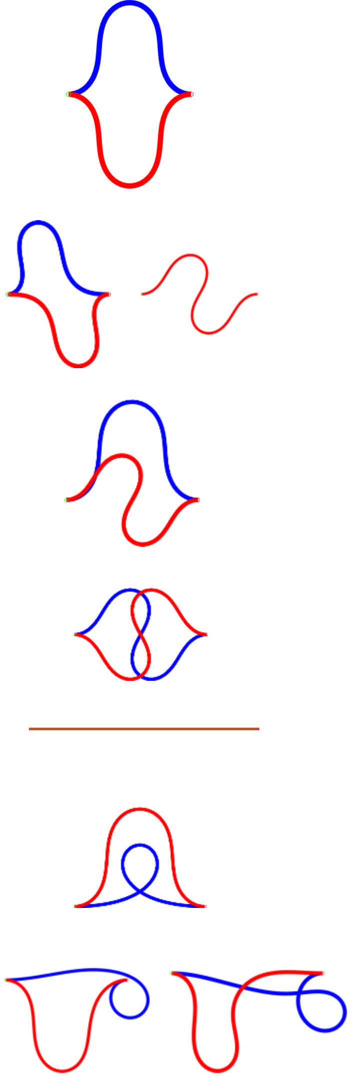

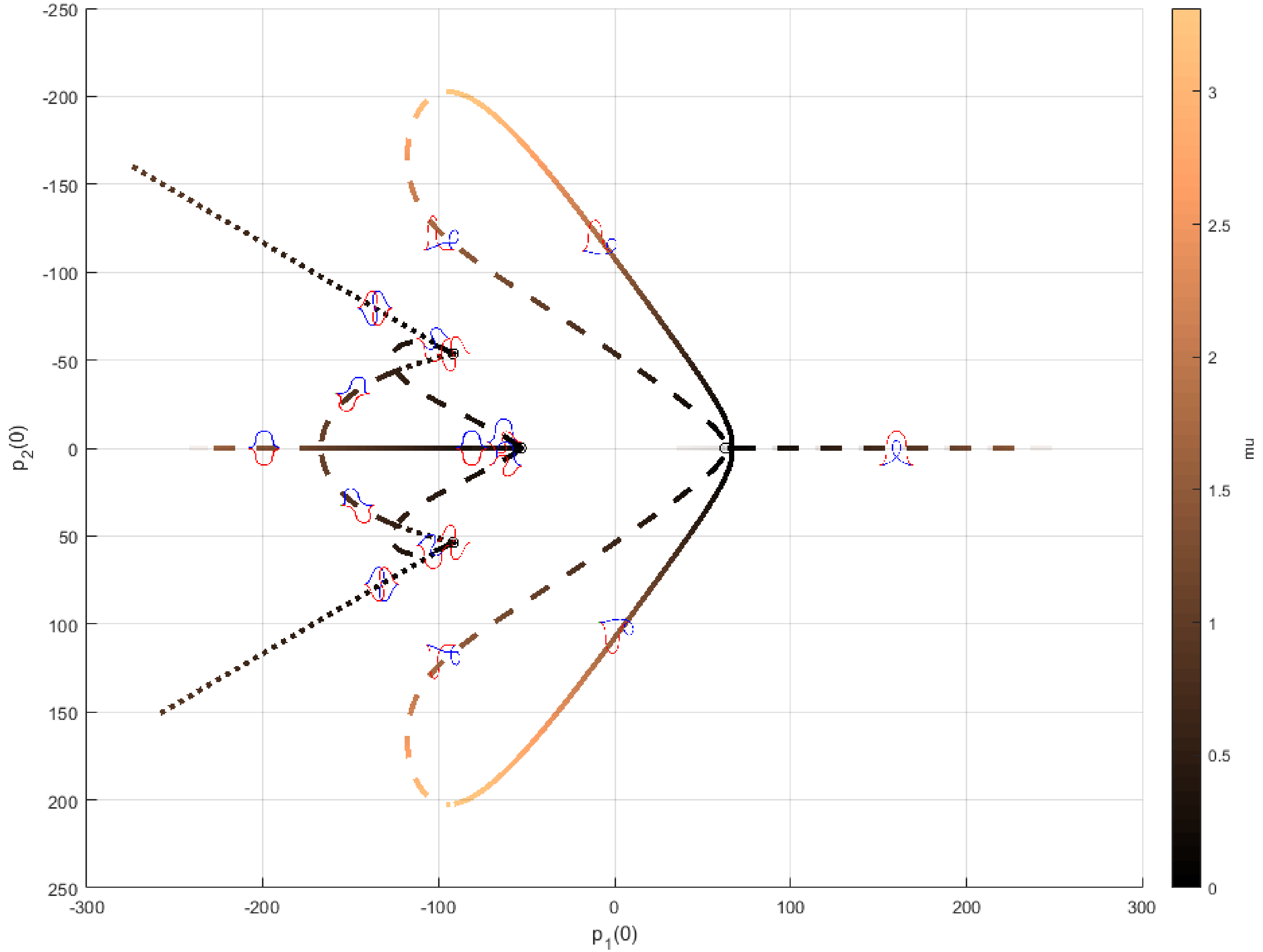

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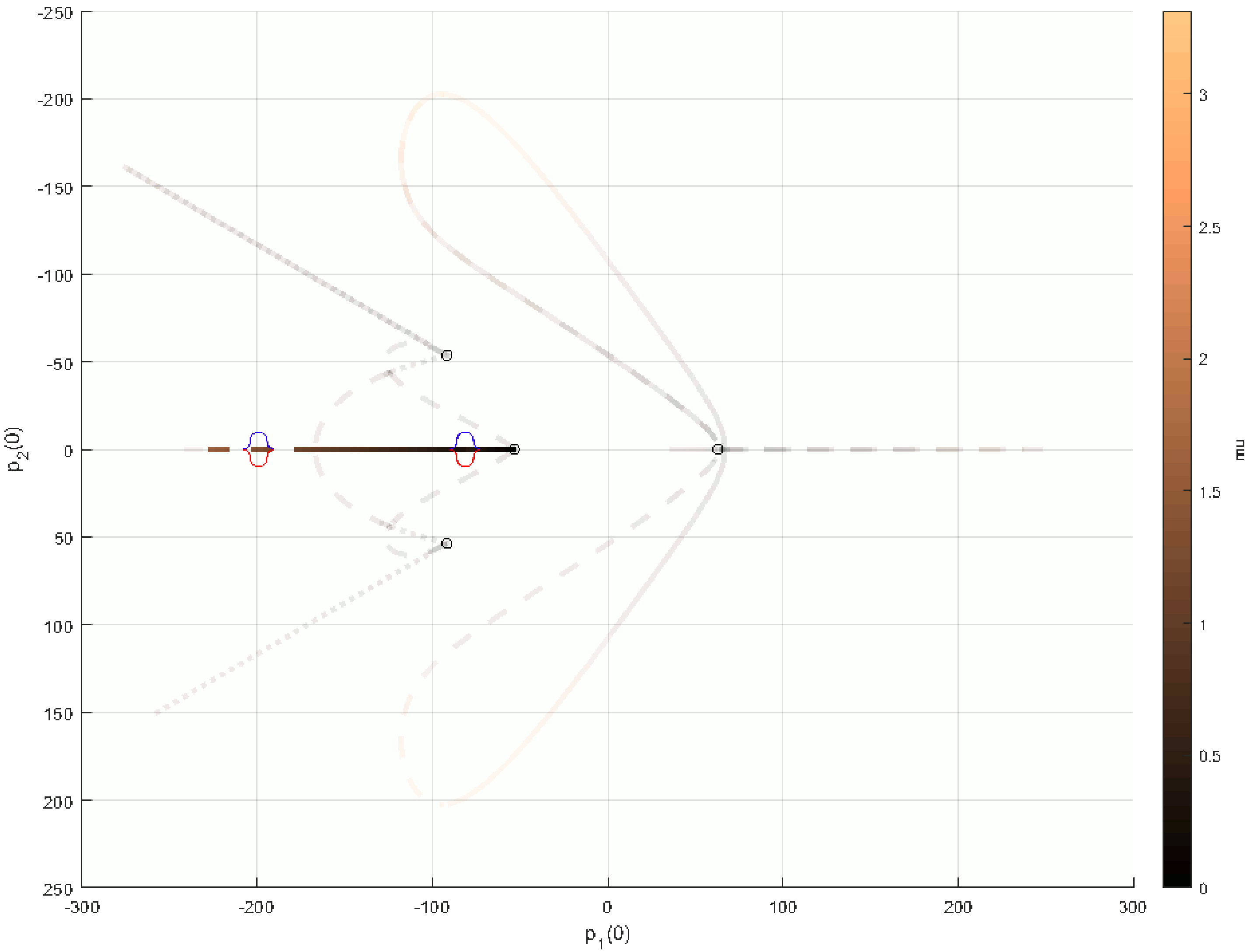
What Does this Plot Mean?

- Insight into shape and optimality of tree for coupling between 0 and ∞
- Connected trajectories represent solutions which can be moved between using continuous deformation



Recapitulation

Recapitulation

Multiple optimal solutions exist to our problem, with interesting results in bifurcation land

There are probably more interesting, optimal solutions we haven’t found that *only* exist for larger values of μ

Provides insight to cases where there are more than two agents

References

Justh EW, Krishnaprasad PS. 2015 Optimality, reduction and collective motion. *Proc. R. Soc. A* 471:20140606. http://dx.doi.org/10.1098/rspa.2014.0606